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STRONGLY MINIMAL GENERALIZED CLOSED SET IN BIMINIMAL STRUCTURE SPACES

[a]E.Subha and [b]N.Nagaveni

[a]Department of Science and Humanities, Karpagam College of Engineering, Coimbatore-32, Tamilnadu, India.

[b]Associate Professor in Mathematics, Coimbatore Institute of Technology,Coimbatore-14
E-mail: prajeethsubha@gmail.com

Abstract

In this paper, we introduce the concept of smg-closed sets in biminimal structure space and a new notion of a pair wise smg-closed set is defined and studied some of its properties.

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1.Introduction

The study of bitopological spaces was first initiated by Kelly[4] and Fututake[3] introduces generalized closed sets in bitopological spaces. Noiri[6] introduced the notion of generalized m-closed(briefly mg-closed) set and unified certain types of modifications of g-closed sets. C.Boonpok[1] introduced the concept of biminimal structure spaces and studied $m_X^{\ 1} m_X^{\ 2}$ - closed sets and $m_X^{\ 1} m_X^{\ 2}$-open sets in biminimal structure spaces. C.Boonpok et.al[2] introduced gm (i,j)-closed sets, m (i,j)- $T_{\frac{1}{2}}$ spaces and gM (i,j)-continuity for biminimal structure spaces and investigated some of their properties.

In this paper, we introduce (i,j)-smg-closed set, (i,j)-mg-closed set in biminimal structure space and studied some of their properties.

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2.Preliminaries

We recall the following definitions which are prerequisite for this paper.

Definition 2.1:[5] A subfamily m_X of the power set $P(X)$ of a nonempty set X is called a minimal structure(briefly m-structure) on X if $\phi \in m_X$ and $X \in m_X$

By (X, m_X), we denote a non-empty set X with a minimal structure m_X on X and call it an m-space. Each member of m_X is said to be m_X-open and the complement of a m_X-open set is said to be m_X-closed.

Definition 2.2:[5] Let X be a non empty set and m_X an m-structure on X. For a subset A of X, the m_X closure of A and the m_X interior of A are defined in [9] as follows:

(1) $m_X - cl(A) = \cap \{\mathrm{F} : \mathrm{A} \subset \mathrm{F}, \mathrm{X} - \mathrm{F} \in m_X\}$

(2) $m_X - \mathrm{int}(A) = \cup \{\mathrm{F} : \mathrm{U} \subset \mathrm{A}, \mathrm{U} \in m_X\}$

Definition 2.3:[5] A minimal structure m_X on a nonempty set X is said to have property B if the union of any family of subsets belong to m_X.

Lemma 2.4:[5] Let X be a nonempty set and m_X a minimal structure on X satisfying property B. For a subset A of X, the following properties hold:

(i) $\mathrm{A} \in \mathrm{m_X}$ if and only if $m_X - \mathrm{int}(A) = A$

(ii) A is m_X-closed if and only if $m_X - cl(A) = A$

(iii) $\mathrm{m_X} - \mathrm{int}(A) \in m_X$ and $m_X - cl(A)$ is m_X-closed.

Definition2.5:[6] Let (X, m_X) be an m-space. A subset A of X is said to be minimal generalized closed(mg-closed) if $m_X - cl(A) \subseteq G$ whenever A $\subseteq$ G and G is m_X-open.

Definition 2.6:[7] Let (X, m_X) be an m-space. A subset A of X is said to be strongly minimal generalized closed(smg-closed) if $m_X - cl(A) \subseteq G$ whenever A $\subseteq$ G and G is mg-open. The complement of an smg-closed set is called a smg-open set in (X, m_X).

Definition 2.7:[6] Let $f : X \rightarrow Y$ be a function, where X is a nonempty set with a minimal structure m_X and Y is a topological space. The function $f : X \rightarrow Y$ is said to be $m-continuous$ if for each $x \in X$ and each open set V of Y containing $f(x)$,there exists a subset $U \in m_X$ containing x such that $f(U) \subset V$

Definition 2.8: [1]Let X be a nonempty set and $m_X{}^1$, $m_X{}^2$ be minimal structures on X. A triple $(X, m_X{}^1, m_X{}^2)$ is called a biminimal structure space(briefly m-space).

3. (i, j)-smg-closed set in Biminimal Structure Space

In this section, we introduce (i, j)-mg-closed, (i, j)-mg-open, (i, j)-smg-closed, (i, j)-smg-open, P-smg-closed in biminimal structures and investigate their properties in biminimal structure space.

Definition 3.1: A subset A of a biminimal structure space (X, m_1, m_2) is called (i, j)- minimal generalized closed (briefly (i, j)-mg-closed) if $m_j - cl(A) \subset U$ whenever $A \subset U$ and U is m_i-g-open in X where i, j =1,2 and $i \neq j$.

Definition 3.2: A subset A of a biminimal structure space (X, m_1, m_2) is called (i, j) -strongly minimal generalized closed (briefly (i, j)-smg-closed) if $m_j - cl(A) \subset U$ whenever $A \subset U$ and U is m_i -mg-open in X where i, j =1,2 and $i \neq j$.

Definition 3.3: A subset A of a biminimal structure space (X, m_1, m_2) is said to be pairwise smg-closed (briefly P-smg-closed) if A is (1,2)-smg-closed and (2,1)-smg-closed.The complement of a pairwise smg-closed set is said to be pairwise smg-open(briefly P-smg-open).

Remark 3.4: By setting $m_1 = m_2$ in definition 3.2, a (i, j)–smg -closed becomes smg-closed set.

Definition 3.5: A subset A of a biminimal structure space (X, m_1, m_2) is said to be pairwise m_X -closed (briefly P- m_X -closed) if A is (1,2)- m_X -closed and (2,1)- m_X -closed.The complement of a pairwise m_X -closed set is said to be pairwise m_X -open(briefly P- m_X -open).

Theorem 3.6: Every m_j -closed set in X is (i, j)–smg- closed in X .

Proof: Let $A \subset X$ be m_j -closed and $A \subset U$ such that U is (i, j)–smg- open. Then $m_j - cl(A) = A$ and so $m_j - cl(A) \subset U$. Thus A is (i, j)– smg- closed.

Remark 3.7: The converse of the theorem need not be true as seen from the following example.

Example 3.8: Let X= {a,b,c},m_1={ ϕ ,X,{a},{a,b}} and m_2={ ϕ ,X,{c},{a,c}}. The set A = {a,c} is (1,2) –smg -closed but not 2- closed.

Theorem 3.9: Every (i, j)–smg -closed set in X is an (i, j)-mg -closed set in X.

Proof: Let A be an (i, j)-smg -closed set. Let $A \subseteq U$ where U is P- m_X -open. Since every P- m_X -open set is (i, j)-mg –open and A is (i, j)–smg –closed, we have $m_j - cl(A) \subset U$. Therefore A is (i, j)–mg –closed.

Remark 3.10: The converse of the theorem need not be true as seen from the following example

Example 3.11: Consider the space X={a,b,c} with m_1={ ϕ ,X,{a}} ,m_2={ ϕ ,X} . The set {a,b} is (1,2)-mg -closed but not (1,2) –smg –closed set.

Theorem 3.12: Union of two (i, j)-smg –closed sets in X is (i, j)-smg –closed in X.

Proof: Let A, B $\subset$ X be (i, j)–smg –closed sets. Let U be an (i, j)-mg –open subset of X such that AUB $\subset$ U . We have $m_j - cl(A) \cup m_j - cl(B) \subset U \cup U = U$, since A and B are (i, j)-smg-closed. Hence AUB is (i, j) -smg-closed.

Theorem 3.13: A subset A of X is (i, j)-smg-closed if and only if $m_j - cl(A) \setminus A$ contains no non-empty (i, j)-mg-closed set in X.

Proof: Let F be an (i, j)–mg – closed subset of $m_j - cl(A) \setminus A$ .Now F$\subset$ $m_j - cl(A) \setminus A$ and A$\subset$X\F where A is (i, j)-smg-closed and X\F is (i, j)–mg –open. Thus $m_j - cl(A) \subset$X\F or equivalently F$\subset$X\ $m_j - cl(A)$ .By assumption, we have that F$\subset$ $m_j - cl(A)$ and so F$\subset$ ($X \setminus m_j - cl(A) \cap m_j - cl(A)$)=$\phi$. This shows that F is empty.

Conversely, assume $m_j - cl(A) \setminus A$ contains no non-empty (i, j)-mg -closed set .Let A$\subseteq$U, U is (i, j) -mg-open. Suppose that $m_j - cl(A)$ is not contained in U. Then $m_j - cl(A) \cap U^c$ is a nonempty (i, j)–mg-closed set of $m_j - cl(A) \setminus A$ which is a contradiction. Therefore $m_j - cl(A) \subset U$ and hence A is (i, j)smg-closed.

Theorem 3.14: If a subset A of X is (i, j)-smg-closed and $A \subseteq B \subseteq m_j - cl(A)$,then B is (i, j)-smg-closed in X.

Proof: let B$\subset$U where U is (i, j)-mg-open. Since A is (i, j)-smg-closed and A$\subset$B, it follows that A$\subset$U. By hypothesis $B \subset m_j - cl(A)$ and hence $m_j - cl(B) \subset m_j - cl(A) \subset U$. Thus B is (i, j)– smg –closed.

Definition 3.15: A subset A of a biminimal structure space (X, m_1, m_2) is called (i, j)- minimal generalized open set (briefly (i, j)-mg-open)) if A^c is (i, j)-mg –closed.

Definition 3.16: A subset A of a biminimal structure space (X, m_1, m_2) is called (i, j) -strongly minimal generalized open set (briefly (i, j)-smg-open)) if A^c is (i, j)-smg –closed.

Theorem 3.17: A subset A of X is (i, j)-smg –open in X if and only if F$\subset$ $m_j - \mathrm{int}(A)$ whenever F$\subseteq$A and F is (i, j)-mg-closed in X.

Proof: Let A be (i, j)-smg - open and suppose F$\subset$A where F is (i, j)-mg-closed .Then X\A is (i, j)-smg-closed and X\A $\subset$ X\F, where X\F is (i, j) -mg-open set. This implies that $m_j - cl(X \setminus A) \subset X \setminus F$. Now $m_j - cl(X \setminus A) = X \setminus m_j - \mathrm{int}(A)$.Hence $X \setminus m_j - \mathrm{int}(A) \subset X \setminus F$ and F$\subset$ $m_j - \mathrm{int}(A)$.

Conversely, if F is an (i, j)-mg-closed set with F$\subset$ $m_j - \mathrm{int}(A)$ whenever F$\subset$A. Then X\A$\subset$X\F and X\ $m_j - \mathrm{int}(A) \subset$X\F. Thus $m_j - cl(X \setminus A) \subset$X\F . Hence X\A is (i, j)-smg-closed and A is (i, j)-smg –open.

Theorem 3.18: For each x$\in$X, {x} is -mg-closed in X or $\{x\}^c$ is (i, j)-smg-closed in X.

Proof: If {x} is not (i, j)-mg –closed ,then the only m_j –mg-open set containing $\{x\}^C$ is X. Thus $m_j - cl(\{x\})^c$ $\subset$X and $\{x\}^c$ is (i, j)-smg –closed.

Lemma 3.19: If A and B are two (i, j)-smg-open subset of a biminimal space X, then A∩B is (i, j)-smg-open.

Proof: Suppose that F is (i, j)-mg-closed set contained in A∩B .Since A and B are (i, j)-smg –open sets , then by theorem 3.17,F $\subset$ $m_j - \text{int}(A)$ and F $\subset$ $m_j - \text{int}(B)$. Thus F $\subset$ $m_j - \text{int}(A)$ $\cap m_j - \text{int}(B)$ = $m_j - \text{int}(A \cap B)$.Hence F$\subset$ $m_j - \text{int}(A \cap B)$ and therefore A∩B is (i, j)-smg –open.

Proposition 3.20: Let m_1 *and* m_2 be minimal structures on X satisfying property B. If A is a (i, j)–mg- closed set of (X, m_1, m_2), then $m_i - cl(\{x\}) \cap A \neq \phi$ holds for each x$\in$ $m_j - cl(A)$where i, j =1,2 and $i \neq j$.

Proof: Let x$\in$ $m_j - cl(A)$.suppose that $m_i - cl(\{x\}) \cap A \neq \phi$. Then A$\subseteq$X$-(m_i - cl(\{x\}))$.Since A is (i, j) -mg-closed and X $-(m_i - cl(\{x\}))$ is m_i – open, $m_j - cl(A)$ $\subseteq$ X $-(m_i - cl(\{x\}))$. Consequently, $m_j - cl(A) \cap$ $m_i - cl(\{x\})$. This is a contradiction.

Proposition: 3.21: Let A and B be subsets of a biminimal structure space (X, m_1, m_2) such that $m_j - \text{int}(A) \subseteq B \subseteq A$ .If A is (i, j)-mg-open then B is (i, j)-mg -open ,where i, j =1,2 and $i \neq j$

Proof: Suppose that A and B be subsets such that $m_j - \text{int}(A) \subseteq B \subseteq A$. Let A be (i, j)–mg-open . Let $F \subseteq B$and F is m_i -closed. Since $F \subseteq B$ and $B \subseteq A$, we have $F \subseteq A$.Therefore F $\subseteq m_j - \text{int}(A)$. Since $m_j - \text{int}(A) \subseteq B$, we have $m_j - \text{int}(m_j - \text{int}(A)) \subseteq m_j - \text{int}(B)$ .Therefore $m_j - \text{int}(A) \subseteq m_j - \text{int}(B)$ .consequently F $\subseteq$ $m_j - \text{int}(B)$.Hence B is (i, j)-mg-open.

Proposition 3.22: Let A be a subset of a biminimal structure space(X, m_1, m_2).If A is (i, j)-mg closed, then $m_j - cl(A) - A$ contains no non-empty m_i – closed set, where i, j =1,2 and $i \neq j$.

Proof: Let A be an (i, j)-mg closed set and F is a m_i - closed set such that F$\subseteq$ $m_j - cl(A) - A$. Since A $\in (i, j) - mg(X)$,we have $m_j - cl(A) \subseteq X - F$. Thus, F$\subseteq m_j - cl(A) \cap (X - (m_j - cl(A))) = \phi$

Proposition 3.23: If a subset A of a biminimal structure space (X, m_1, m_2) is (i, j) -mg-closed, then $m_j - cl(A) - A$ is (i, j)-mg-open, where i, j =1,2 and $i \neq j$.

Proof : Suppose that A is (i, j)-mg-closed .We shall show that $m_j - cl(A) - A$ is (i, j)-mg-open .Let F$\subseteq$ $m_j - cl(A) - A$and F is m_i - closed .Since A is (i, j)-mg-closed ,we have $m_j - cl(A) - A$ does not contain non-empty m_i -closed by proposition 3.22. consequently, F $\neq \phi$ .Therefore ϕ $\subseteq$ $m_j - cl(A) - A$, $\phi \subseteq m_j - \text{int}\ (m_j - cl(A) - A)$, we obtain F$\subseteq$ $m_i - \text{int}$ ($m_j - cl(A) - A$) .Hence $m_j - cl(A) - A$ is (i, j)-mg-open.

References

[1] Chawalit Boonpok, Biminimal structure spaces, Int.Math.Forum, 5(15) (2010), 703-707.

[2] Chokchai Viriyapong, Maliwan Tunapan,Witchaya Rattanametawee, Chawalit Boonpok , Generalized m-closed sets in Biminimal structure spaces, Int. Journal of Math . Analysis, Vol.5, 2011, no.7, 333-346.

[3] Fututake, On generalized closed sets in bitopological space, Bull. Fukuoka. Univ. Ed. part III, 35 (1985), 19-28.

[4] Kelly.J.C, Bitopological spaces, Proc. London Math.Soc.(3),13 (1963),71-89.

[5] Noiri.T, 11th meetings on topological spaces and its Applications,Fukuoka University Seminar House,2006, 1-9.

[6]Noiri.T and V.Popa," A Decomposition of m-continuity" Seria Matematica-Informatica- Fizica,Vol.LXII,No.2/2010, 46-53.

[7] Pushpalatha.A and Subha.E, Strongly Generalized closed sets in Minimal structures,Int.Journal of Math. Analysis Vol.3, 2009, no.26, 1259-1263.